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478. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Solve the equations, $l/x + y/m + z/n = 1$, $x/l + m/y + z/n = 1$, $x/l + y/m + n/z = 1$.

SOLUTION BY HENRY D. THOMPSON, Princeton N. J.

Take X, Y, Z as symbols for x/l, y/m, z/n, respectively. Then the equations to be solved are:

$$(1)\ 1/X + Y + Z = 1, \qquad (2)\ X + 1/Y + Z = 1, \qquad (3)\ X + Y + 1/Z = 1.$$

The difference of (2) and (1) gives (4) $(X - Y) + (X - Y)/XY = 0$, one solution of which is (5) $X = Y$. This substituted in (1), (2), (3) leaves to be solved (6) $1/X + X + Z = 1$, (7) $2X + 1/Z = 1$, and (7) gives $Z = 1/(1 - 2X)$.

This in (6) gives the cubic

$$1/X + 1/(1 - 2X) = 1 - X, \qquad \text{or} \qquad (1 - X)/X(1 - 2X) = (1 - X),$$

the solutions of which are $X = 1$, and $X = 1/4 \pm \sqrt{-7}/4$. These with (5) and (7) give for $[x, y, z]$ the three solutions:

$$(\text{I, II})\ [(1 \pm \sqrt{-7})l/4,\ (1 \pm \sqrt{-7})m/4,\ (1 \pm \sqrt{-7})n/4], \qquad (\text{III})\ [l, m, -n].$$

The other solution of (4) viz.: (8) $1 + 1/XY = 0$ in (1) or (2) gives $Z = 1$, whence (3) becomes $X + Y = 0$, which with (8) gives $X^2 - 1 = 0$, and $X = 1$ with $Y = -1$, or $X = -1$ with $Y = 1$. Two other solutions are:

$$(\text{IV})\ [l, -m, n], \qquad (\text{V})\ [-l, m, n].$$

To find the infinite solutions, set X/W, Y/W, Z/W, respectively, for X, Y, Z in (1), (2), (3) and these equations rid of denominators, and with $W = 0$ are:

$$(9)\ X(Y + Z) = 0, \qquad (10)\ Y(Z + X) = 0, \qquad (11)\ Z(X + Y) = 0.$$

The equation (9) is satisfied when $X = 0$, which set in (10) or (11) gives either $Y = 0$ or $Z = 0$. The other solution of (9) viz.: (12) $Y = -Z$ set in (10) and (11) gives $Y(X - Y) = 0$ and $-Y(X + Y) = 0$; whence $Y = 0$, and from (12), $Z = 0$. Thus the infinite solutions (VI), (VII), (VIII) are the points at infinity on the axes when x, y, z are taken as coördinates in a rectilineal system.

Note that each of the three hyperbolic cylinders (1), (2), (3) has for its asymptotic planes a coördinate plane and a plane parallel to the like named axis.

Also solved by B. J. DINING, GERTRUDE I. MCCAIN, J. L. RILEY, E. F. CANADY, S. W. REAVES, J. Q. MCNATT, G. W. HARTWELL, R. A. JOHNSON, C. H. WORTHINGTON, PAUL CAPRON, HORACE OLSON, and G. Y. SOSNOW.

479. Proposed by S. A. COREY, Albia, Iowa.

Prove or disprove

$$\left\{ \begin{vmatrix} x & -v & -z \\ -y & -z & v \\ -z & y & -x \end{vmatrix}^2 + \begin{vmatrix} y & -v & -z \\ x & -z & v \\ v & y & x \end{vmatrix}^2 + \begin{vmatrix} x & y & -z \\ -y & x & v \\ -z & v & -x \end{vmatrix}^2 + \begin{vmatrix} x & -v & y \\ -y & -z & x \\ -z & y & v \end{vmatrix}^2 \right\}$$

$$\div \begin{vmatrix} x & -y & -z & v \\ y & x & -v & -z \\ z & v & x & y \\ v & -z & y & -x \end{vmatrix}^2 = (x^2 + y^2 + z^2 + v^2)^{-1}.$$

SOLUTION BY HENRY D. THOMPSON, Princeton, N. J.

Let Q be a symbol for $(x^2 + y^2 + z^2 + v^2)$, and represent the determinants in the order in which they appear in the equation by Z, V, X, Y, H, so that the equation is

$$\{Z^2 + V^2 + X^2 + Y^2\} \div H^2 = Q^{-1}.$$